A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING
The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

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The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation and Equity*, respectively.

*JMETC* readers are educators from pre-K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

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Toward an Analysis of Video Games for Mathematics Education

Kathleen Offenholley
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Video games have tremendous potential in mathematics education, yet there is a push to simply add mathematics to a video game without regard to whether the game structure suits the mathematics, and without regard to the level of mathematical thought being learned in the game. Are students practicing facts, or are they problem-solving? This paper examines several schema for assessing how the structure of a video game interacts with the mathematical content. The schema include whether the mathematics is intrinsic to the structure of the game, and whether the game is epistemic, that is, whether players take on the identity of a mathematician or problem-solver while playing. Implications for future video games are discussed.

Keywords: games, video games, computer games, mathematics, mathematics education, gaming, middle school, high school, college.

Introduction

Nearly as long as computers have been able to play games and simulate reality, educators have been attempting to tap that potential to teach students. Seymour Papert, in his groundbreaking book *Mindstorms: Children, Computers and Powerful Ideas* (1980), showed how programming the Logo turtle could help children learn to think logically through exploration and manipulation of a computer-simulated object. In the intervening years, computer technology has improved exponentially, yet where computers are used most powerfully to open up stimulating adventures and challenges is not in the classroom: it is in the home, with video games.

Video games are a common part of nearly all (97%) of teens’ lives, with most playing on a regular basis, according to a 2008 study by Lenhart, Kahne, Middaugh, Macgill, Evans, and Vitak, sponsored by the Pew Internet and American Life Project. This study also found that video games are played by both girls and boys, and across all socio-economic backgrounds, thus dispelling the stereotype of the isolated male game player.

How Video Games Work to Promote Learning

Our students, who groan at the thought of an hour of homework, or who give up in dismay after only a few minutes of work on a difficult math problem, often go home to spend hours playing complex video games that involve constant calculating, planning, and problem solving. Because video games strive to maintain a fun or engaging atmosphere, players may be more receptive to working through problems that are quite difficult or even tedious. In the language of cognitive psychology, “…video games increase activation and arousal, which may improve task performance” (Schmidt & Vandewater 2008, 64). Players are willing to put in the hours and hours of time that cognitive psychology tells us it takes to move from being a novice to becoming an expert (see for example, Anderson 2000; or for a more popular description, Gladwell 2008). Moreover, according to James Gee, author of the ground-breaking, *What Video Games Have to Teach Us about Learning and Literacy* (2007), video games dole out information at a user-determined pace, so that the learner can acquire knowledge as it is needed.

In addition to the acquisition of facts and skills through repeated practice, players often engage in deep-level problem-solving. As Mayo notes in *Science* (2009, 80), “Game-based tasks often require the formation of hypothesis, experimentation, and discovering the consequences of actions taken.” Trial and error, a problem strategy that my students are often loath to use, is key in gaming. Making mistakes is a large part of how players learn, the game principle Gee calls “failing forward.” In school, it is often bad to get a wrong answer, or to fail at a task; in video games, it is expected.

Mathematician Keith Devlin (2011) contends that most of Gee’s 36 principles of how learning takes place in games “are exactly what it takes to achieve the five strands of…mathematical proficiency,” (84) namely, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition.

Finally, video games are a powerful tool for learning because they simulate reality, allowing us to create imaginary worlds and identities, according to David Shaffer in *How Computer Games Help Children Learn* (2006). A game player can experience being a businessperson, not just by imagining that she is one, but by immersing herself in the language, structure, and actions of that field—unlike the static description in a mathematical word problem. Such immersion, says Shaffer, endows the work the player does with a deeper, epistemological meaning. Devlin (2011) sees a great...
potential for students to do mathematics and to be mathematicians within the context of a game, adopting the identity of a person who is mathematically able.

What Makes a Good Video Game?

From the description above, it might seem that there is an enormous potential for video games to help students learn in the classroom. When we try to turn activities that are meant to be fun into activities that are meant primarily to teach, some of the best parts can become lost in that translation. There is also so much hype surrounding the potential for video games for education, it is imperative to sort out what really works. As Gunter, Kenny, and Vick warn, “We are witnessing a mad rush to pour educational content into games in an ad hoc manner in hopes that player/learners are motivated simply because the content is housed inside a game.” (2008, p. 511)

A meta-analysis of gaming studies show that educational computer games in general can have significant learning gains and improvements in attitude over traditional teaching (Vogel, J. F., Vogel, D. S., Cannon-Bowers, J., Bowers, C. A., Muse, K. & Wright, M., 2006). However, in an article for the Naval Training Division¹, Robert T. Hays (2005) analyzes 105 articles on instructional games, and has an important caveat that proponents of educational gaming would be wise to heed. He warns that, “The empirical research on the effectiveness of instructional games is fragmented,” because of the different ages, games, and learning tasks. “We should not generalize from research on the effectiveness of one game in one learning area for one group of learner to all games in all learning areas for all learners.”

To truly analyze the effect of games-based learning, we must begin to examine rigorously what each game does for its particular group of learners, and for the particular mathematics we hope it will impart. We would like to be able to answer the questions: What does the game teach, and on what level of knowledge? Are the game’s mechanics the best ones to convey the mathematics and to capture our students’ attention?

To answer the first question, we can use standardized tests to measure educational outcomes, but for an examination of deeper knowledge construction, it may be more revealing to see results of mind maps and interviews, including recording of student reactions as they solve problems.

The second question may be difficult to answer for those without a background in game design; lack of attention to this question may account for the poor quality of many current educational video games. Scott McLeod’s now infamous 2009 blog post, “Do Most Educational Games Suck?” pointed to lack of complexity and poor graphics as just some of the problems he saw. (See Ackerman, Kong & Desiato 2011 for an analysis of the responses to his blog.) It is a difficult question to answer, because there are so many ways to analyze how a game works. Pioneer game theorists Salen and Zimmerman (2003) explain that a game can be analyzed by looking at the rules (the formal design schemas), play (the players’ experience of the game) or culture (the larger cultural contexts surrounding and embedded within the game). For the purposes of this paper, we will confine ourselves only to the first area, the rules of the game, and how that structure interacts with the mathematics taught. Other aspects of design are important but are beyond the scope of this paper.

Several schemas have been used to examine how structure interacts with learning in a game. These include whether the learning is extrinsic or intrinsic to the game, whether the learning is an integral part of a storyline, or fantasy, and whether the game is epistemic. We shall see that these schemas are intertwined, each attempting to get at something similar within the game.

Intrinsic and extrinsic motivation in psychology have to do with whether a person is motivated internally, or by external rewards. Malone and Lepper (1987) use this concept to describe the structure of a game. In an intrinsic game, the concepts being learned are an integral part of the game; in an extrinsic game, different concepts can be taught in the same game structure without changing the game. Devlin theorizes that for a mathematics game to be good, “…the math should arise naturally in the game, and it should have meaning in the game” (p. 127), that is, it should be intrinsic.

To illustrate the difference between an intrinsic and extrinsic game, I present two very different video games, both for middle school mathematics, Dimension M. and Ko’s Journey.

In April 2010, a New York TV news channel reported excitedly, “Thousands of city students have become hooked on Dimension M, a role-playing video game that requires quick math skills to keep going” (Christ 2010). In Dimension M. students navigate a new planet as space explorers. Their mission is to capture correct answers to math questions and accrue points. Learning takes place outside of game play, with the game is like a high-speed quiz or drill of already learned concepts. This game is extrinsic—any questions can be used with the same game mechanics.

In the game Ko’s Journey, players follow the adventures of a girl who has been separated from her family and must travel across the wilderness. Along the way, she must solve problems using mathematics, including finding the correct proportion of medicinal plants to save a wolf cub, and the correct angle to shoot her arrows. This game is intrinsic—the particular mathematics of ratio and proportion and angles is an integral part of the

¹ The military, along with the medical profession, has been at the forefront of using simulations and games for training purposes. See for example, the America’s Army game series.
game, and we cannot swap out this content for different content, without having to alter the game.

It must be noted that with these two particular games, there is no evidence that one kind of game structure trumps another in terms of learning. Large-scale studies of Ko’s Journey have not been done. Dimension M has gotten favorable results in terms of post-test differences between the treatment group who played Dimension M, and the control group who did not, based on scores on the district-wide exam (Kebritchi, Hirumi & Bai, 2010). However, the study did not look at measures of deeper learning using interviews, mind maps, or other methods. In addition, there was no control group practicing the same concepts with paper and pencil for the same amount of time—the control group did nothing. A study of a similar game, Nintendo’s Brain Training, had the control group do pencil and paper puzzles and showed no significant gains (Sage, 2009). To be sure, Dimension M is more fun than paper and pencil, so students may keep playing it more than they might with only pencil and paper. However, the game does not require the formation and testing of hypothesis, or any other kind of deeper problem solving.

We are left to conjecture that Ko’s Journey may help students get better at using mathematics to solve problems, while Dimension M may help students get better at taking standardized tests without improving general problem solving skills. At the moment, however, this is only conjecture.

Gunter, Kenny and Vick (2008) have proposed a model called RETAIN for analyzing educational games that is very similar to the intrinsic/extrinsic model we have just examined. “The targeted content,” they contend, “needs to be intrinsically coupled with the fantasy context (or story, if one exists) of the game” (p. 517). Players should acquire knowledge in a natural way within the game, so that the knowledge can transfer to new situations.

Using this model, a panel of game designers, instructional designers and education researchers scored Math Blaster only 18 out of 63 on their rubric (by contrast, Where in the world is Carmen Sandiego? received 41 out of 63). Math Blaster can be thought of as a forerunner to Dimension M., and is similar in design. Their critique of the game: “Math Blaster needs to create more opportunity for naturalization and transfer, an unfortunate anomaly created by the fact that the game is so focused towards improving a very specific and focused set of skills” (p. 531).

The RETAIN model would likely give Ko’s Journey high marks for integrating mathematics with a fantasy/story line, and for learning that arises naturally within the context of the game.

Another schema for examining the intersection between gaming and learning is whether the video game recreates what Shaffer (2006) calls the epistemic frame of a discipline: the unique technical language and symbols of the discipline and the way that an expert of that discipline sees the world. An epistemic game is one which requires the player to think using the rules and problem solving strategies of that frame. Both Shaffer (2006) and Devlin (2011) posit that epistemic games offer better vehicles for deeper learning.

One such game is NIU-Torcs, a simulation game created specifically for college mathematics education. Brianno Coller and colleagues developed the game through an NSF grant to help their mechanical engineering students learn numerical methods (Coller & Scott, 2009). Students begin the game by learning how to code acceleration and steering using the programming language C++. As with any good game, these first tasks are relatively easy. According to the authors, “We have invited high school students onto campus to play the game; they are able to do it, usually within an hour or two. Making the car move fast and nimbly without skidding off the road, however, is a challenge that takes nearly fifteen weeks to fully realize” (p. 902). To do the latter, students must calculate numerical roots, solve systems of linear equations, and be able to do curve fitting and simple optimization. The authors report that students are motivated to keep trying far more than when given these types of problems as meaningless homework exercises. Moreover, the students are involved in authentic work that allows them to begin to think and act like real engineers.

Although measures of low-level knowledge were statistically identical, concept maps produced by the students in both the game-based and traditional classes showed that students in the game-based class had much greater levels of deep thinking, which included being able to compare and contrast methods and link concepts together. In addition, student attitudes had changed—the game players were more engaged, and more able to recognize the value of the mathematics they were doing. This last is the “favorable disposition” that Devlin talks about as being so important to learning mathematics.

In addition to being an epistemic game, NIU-Torcs is an intrinsic game: we can only use it to teach this particular kind of mathematics. Would it also score highly in the RETAIN model? It is hard to say without convening Gunter, Kenny and Vick’s panel of experts to review the game, but it must be noted that one of the key elements to their model, a fantasy or story line, is absent. In NIU-Torcs, players are imagining themselves as engineers without the need for an immersive fantasy to set the scene.

Conclusion

This paper has examined three very similar ideas of what makes a good mathematics video game. The key elements of each schema as they pertain to mathematics games are summarized in Figure 1.

Using these schemas, we can begin to imagine an ideal game in which the mathematics is an integral part of a story line, a fantasy of some kind in which the player


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The “theme” of the fall issue of the Journal of Mathematics Education at Teachers College will be Evaluation. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to evaluation in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by January 21, 2012. The spring issue’s guest editor, Ms. Heather Gould, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by February 1, 2012, and final drafts of selected papers are to be submitted by March 1, 2012. Publication is expected by April 15, 2012.

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