TABLE OF CONTENTS

PREFACE

v Matthew DeGraaf, Teachers College Columbia University
Simonne Salmon-Nembhard, Teachers College Columbia University

ARTICLES

1 Rainforest Mathematics
Jeremy Kilpatrick, University of Georgia

9 Using a Framework of 21st Century Competencies to Examine Changes between China’s 2001 and 2011 Mathematics Curriculum Standards for Basic Education
Max Stephens, University of Melbourne, Australia
Richard Xu Keqiang, South West University, China

17 Solving Optimization Problems with Dynamic Geometry Software: The Airport Problem
José Contreras, Ball State University

29 Using Dynamic Software to Address Common College Calculus Stumbling Blocks
Alice W. Seneres, Rutgers, The State University of New Jersey
John A. Kerrigan, Rutgers, The State University of New Jersey

39 Rousing Students’ Minds in Postsecondary Mathematics: The Undergraduate Learning Assistant Model
David C. Webb, University of Colorado Boulder
Eric Stade, University of Colorado Boulder
Ryan Grover, University of Colorado Boulder

49 Using Mathematics Literature with Prospective Secondary Mathematics Teachers
Christopher C. Jett, University of West Georgia

55 Financial Literacy: An Essential Component of Mathematics Literacy and Numeracy
Marla A. Sole, Guttman Community College, The City University of New York

63 Integrating Universal Design and Response to Intervention in Methods Courses for General Education Mathematics Teachers
Kelley Buchheister, University of South Carolina
Christa Jackson, Iowa State University
Cynthia E. Taylor, Millersville University of Pennsylvania
With ballooning public and private debt, an increasingly complex and diverse array of financial product choices available to individual small investors, a shift in benefit plans that often requires increased individual responsibility, and the growth of unscrupulous predatory lending practices, today’s students need to be financially literate. Americans living in the United States recognize the importance of financial literacy and overwhelmingly believe finance should be incorporated into the United States school’s curriculum. Eighty-nine percent of people who completed the 2012 National Financial Capability Study indicated that finance should be taught in school (Financial Industry Regulatory Authority [FINRA], 2013). Mathematics educational organizations have also published similar recommendations. The National Council of Teachers of Mathematics (NCTM) supports including real-world financial exercises in the curriculum and recognizes the connection between mathematics, numeracy, and financial literacy (NCTM, 2011). The Common Core State Standards for Mathematics (CCSSM) include financial examples in the standards on functions and modeling (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

However, many schools do not devote any time to financial education. Although the number of states requiring financial education has recently increased, to date, only 19 states mandate that K-12 schools offer a course in personal finance (Council for Economic Education [CEE], 2014). A survey of principals in the United States found that 45% of students were unfamiliar with financial principles and concepts. Students found it difficult to estimate answers and judge the reasonableness of their solutions. Reading comprehension and general cognitive skills led students to incorrect answers. The difficulties students had reinforce the need for financial education and developing students’ general cognitive skills.

KEYWORDS financial literacy, numeracy, quantitative reasoning, estimation, debt
personal finance is a topic that has been “greatly neglected,” (p. 13) as it is often not part of the curriculum in traditional mathematics programs.

The purpose of this paper is to illustrate the need for financial education, describe the current level of financial literacy, demonstrate how financial literacy can be taught, and examine why students have low levels of financial literacy. The paper is divided into five sections followed by a conclusion. The first section examines what it means to be financially literate, highlighting how financial literacy connects to mathematical and verbal cognitive abilities. In light of recent economic developments that have shifted financial responsibilities to the individual, the second section justifies the inclusion of financial education in the school curriculum. After discussing why financial literacy is an essential skill that should be taught, the third section examines young adults lack of understanding of finance. Next, mindful of the fact that financial literacy can impact both asset management and equity, the fourth section describes gender and racial differences in the level of financial understanding. The fifth section provides two examples of financial literacy exercises and assesses students’ skill level. Problems emerged as students attempted to make estimates and judge the reasonableness of their answers. Issues related to reading comprehension and general mathematical skills led to incorrect solutions. Given the difficulties students had, the conclusion reiterates the importance of teaching numeracy and financial literacy.

In students’ personal and professional lives and in their roles as active citizens, they will need to make a number of complex financial decisions, many of which will have a lasting impact on their welfare. Providing students the opportunity to work with a variety of real-world financial examples may increase the likelihood that the decisions they make will be financially sound and savvy.

Although the OECD’s definition will be used to frame the discussion, there is a wide range of conceptual and practical definitions that have been offered by other researchers and organizations. Examining other definitions, conceptual frameworks, and measurements used to assess one’s level of financial literacy, the following three key tenets emerge. First, financial literacy requires familiarity with fundamental concepts from economics and finance (FINRA, 2013; Lusardi & Tufano, 2009). Second, financial literacy involves actions, practical experience, and the ability to apply the knowledge one has gained (Jump$tart Coalition for Personal Financial Literacy, 2007; Lusardi, Mitchell & Curto, 2010; Moore, 2003). Thus, financial education without the ability to assess financial options in the real-world and, over one’s lifetime, make sound financial decisions, does not make one financially literate. Third, financial literacy includes knowledge of financial products, which requires knowing how to perform a range of elementary mathematics computations such as calculating simple and compound interest or depreciation (FINRA, 2013; Lusardi & Tufano, 2009).

In order to read financial documents, one must have a degree of familiarity with economic terms and a strong vocabulary, a high level of reading comprehension, and strong mathematical and analytical skills. Hence, it is not surprising that the OECD has found that there is a strong positive correlation between both financial literacy and mathematics and between financial literacy and reading, with correlations of 0.83 and 0.79 respectively (OECD, 2014). Therefore, to ensure that students will be financially literate, teachers need to also develop students’ reading comprehension and mathematics computation skills. Given this understanding of financial literacy, I focus next on why students should have the opportunity to learn about finance in both high school and college.

### Defining Financial Literacy

The OECD defines financial literacy as follows:

Financial literacy is knowledge and understanding of financial concepts and risks, and the skills, motivation and confidence to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the financial well-being of individuals and society, and to enable participation in economic life (OECD, 2014, p. 33).

### Justification for Teaching Financial Literacy

In this section, I discuss how financial education can empower individuals. Given the complex menu of financial products and increased focus on individual responsibility, it is essential that we take steps to ensure sound financial decision making, improve the rate of financial market participation, and lessen risk aversion.

From an early age, students need to understand finance. Not being financially literate makes it difficult for students to navigate college loans and aid applications successfully (Bidwell, 2013). Even if students secure
college aid, loans, and are awarded scholarships, the majority will graduate with some debt. The average amount of college related debt for students who graduated in 2013 was $35,200 (Ellis, 2013). Many students were shocked to learn how much debt they had accumulated, which suggests that they did not have a financial plan (Ellis, 2013). Additionally, parents who want to help finance their children’s college education need to have a financial plan to ensure they have adequately saved and invested (OECD, 2012).

Assuming more independence, young people must know how to budget, assess purchases that will depreciate in value, compare credit card offers, and weigh cellphone plans. As students enter the workforce, increasingly they will need to take responsibility for their retirement savings and healthcare benefits. In the United States, state supported and employer sponsored pension plans are being replaced by plans that individuals are primarily responsible for selecting and funding (Lusardi, 2008; Lusardi, 2012; OECD, 2012). However, although individuals are often responsible for their own retirement, approximately 50% of older Americans in the workforce know little about either their own pension plans or the policies governing how social security benefits will be administered (Lusardi, 2008). Ignorance about financial benefits puts their future at risk.

Outside of work, there are a growing number of investment options that small investors can take advantage of. One option offered to small investors is mutual funds, which pool investors’ money. By 1890, closed-end funds, which are funds that do not issue new shares and do not redeem investors’ old shares, had come to the United States (Rouwenhorst, 2004). By 2012, the number of mutual funds had risen to 7,238 (Silverblatt, 2013). Although the number of options has increased, it is difficult for an individual who is not financially literate to read a financial prospectus and take into account a fund’s fees, goals, rate of return, level of diversification, trading practices, and tax implications to select the best option given the individual’s needs and tolerance for risk. This could be one reason for the low rate of market participation. Presently, only 54% of adults have money in the stock market either in an individual stock, stock mutual fund, self directed 401K or IRA (Jones & Saad, 2014).

The number of high interest loans or predatory loans has also increased. One such loan is a payday loan, which are high interest, short-term, unsecured loans (Hodson, Owens, & Fritts, 2003). Borrowers obtain a small amount of money that they repay to the lender with funds from their next paycheck or next source of income (Hodson et al., 2003). A few decades ago there were no payday loans (Bair, 2005). Today, payday loans are prevalent. If the short-term interest on these loans were converted to an annual percentage rate, the rate would be between 400% and 1,000% (Snarr, 2002). It is unclear if the individuals who take out these short-term loans understand the rate they are paying to borrow money. Researchers have found that victims of predatory lending practices had lower scores on tests measuring their financial acumen compared to the general population (Moore, 2003). Hence, financial illiteracy can be costly.

Last, financial literacy is important for an individual’s role as an active citizen. The United States national debt presently exceeds $17.6 trillion. In 2011, for the first time in history, the failure to rein in spending and reduce the debt led Standard & Poor’s to downgrade the United States AAA credit rating to AA+ (Swann, Chambers, & Beers, 2011). Being financially literate helps one to assess which critical services should be funded, to better understand individual and corporate tax rate policies, financial bailouts, redistribution, and the sharing of risks, while simultaneously addressing how the debt can be reduced.

All this may contribute to financial literacy having been recognized globally as an important life skill. The Group of Twenty (G20), an international organization focused on working together to tackle economic issues and make financial decisions, urged both the World Bank and the Global Partnership for Financial Inclusion (GPFI) to work to foster increased financial education (G20, 2012).

Financial education and strengthening students’ cognitive skills increases financial literacy, which, in turn, can impact financial market participation. One study found that high school students from states that had a mandate which required them to complete some financial education coursework scored significantly higher on a test designed to measure personal financial literacy compared with students from states that did not have such a mandate (Tennyson & Nguyen, 2001). Another study found that although financial education did not impact one’s decision to participate in the financial market, those with higher cognitive abilities were more likely to participate in the financial market (Cole & Shastry, 2008). Given the benefits of financial education and developing students’ general cognitive skills, an important question to investigate is: How financially literate are students?
Young People and Financial Literacy

To measure the level of financial literacy, in 2012, the Program for International Student Assessment (PISA) administered a large-scale test to 15-year-old students from 18 countries (OECD, 2014). The results showed that many students lacked basic financial skills. For example, one question on the test asked students to find how much would have been deposited into a worker’s bank account by examining a pay stub that had the following four dollar amounts: gross salary, net salary, total deductions, and gross salary year to date. Questions were ranked on the level of difficulty on a five point scale, with level five questions being the most challenging. This question was determined to be a level four question (OECD, 2014). Only 31% of students were deemed to be proficient at level four (OECD, 2014).

Many young adults do not understand personal finance. For example, although stocks fluctuate in value, historically, if held for a long time period, they yield greater returns than other more conservative investments such as bonds, savings accounts, and checking accounts. However, only 16.8% of high school seniors and 19.2% of college students thought stocks would outperform these more conservative investments over an 18-year period (Mandell, 2008). On a test designed to measure one’s knowledge of personal finance, college students answered only 53% of questions correctly (Chen & Volpe, 1998).

Despite gaps in their financial knowledge, many young adults have checking accounts and credit cards. However, most do not understand how to reconcile an account or compute interest. A survey of 18-year-olds found that only 43% understand how to balance a checkbook and how to review a bank statement for errors and only 32% know how credit cards calculate fees and interest (Charles Schwab, 2011). A survey of students who were enrolled at either a four-year public or private university found that 29.2% had at least one credit card and 23.7% carried a balance of $1,000 or more (EverFi, 2014). Poor financial decisions made by college students may be the result of not having sufficient knowledge.

Financial Literacy: Gender and Racial Differences

Women and minorities are those most lacking financial knowledge and skills. Exploring gender differences, several researchers have found that women were less financially literate than men (Chen & Volpe, 2002; Lusardi, 2007; Lusardi & Mitchell, 2011; OECD, 2013). However, on one test designed to measure the level of financial literacy of college students, women scored higher than men, with mean scores of 62.6 and 59.7 respectively (Mandell, 2008).

Whichever sex is more financially literate, women tend to be more financially cautious than men. When asked whether they were willing to take financial risks, 24% of men, compared to 11% of women, responded affirmatively (FINRA, 2013). The lack of risk-taking may negatively impact wealth accumulation as, historically, over the long term, less risky investments such as saving accounts and bonds have yielded lower rates of return compared with riskier equity investments. It is concerning that college women were found to be less confident, less enthusiastic, and less willing to increase their knowledge of personal finance compared to college men (Chen & Volpe, 2002).

Exploring racial differences, minorities were found to be less financially literate (Lusardi, 2007; Lusardi & Mitchell, 2011; Mandell, 2008). In college, results from a financial literacy test found that Whites, on average, had higher levels of financial knowledge than Hispanic Americans or African Americans, with mean scores for the three racial categories of 63.3, 59.8, and 56.3 respectively (Mandell, 2008). It is unclear to what extent this gap in scores was influenced by cultural factors, as scores of students from more affluent families or whose parents had a college degree or a more advanced degree were found to be higher (Mandell, 2008).

Lack of knowledge can impact debt management skills. One study found that Whites were less likely than Hispanics or African Americans to engage in costly credit card behaviors such as paying late fees or interest (FINRA, 2013). Comparing the borrowing habits of 18-34 year olds with incomes under $25,000, Whites were less likely than Hispanics or African Americans to use an outlet other than a bank to borrow money (FINRA, 2013).

As life expectancies increase, and public and private supports decrease, women and minorities who are less financially literate are less likely to participate in the financial marketplace, and may, by necessity or lack of knowledge, be more likely to secure high interest loans, and risk outliving their savings.
Financial Literacy: Learning from Students’ Approaches

Given young people’s low performance on tests of financial literacy, I was interested in observing the ways students engage with, and attempt to solve, financial questions. I was particularly interested in students’ familiarity with financial words and figures, their ability to make approximations to answer financial questions, whether they could judge the reasonableness of their answers, and how their mathematics and reading cognitive skills influenced their approach to real-world financial problems.

These lessons were taught to students enrolled in a college Quantitative Reasoning course. The majority of students were Business majors and therefore were assumed to have had some familiarity with, and interest in, finance and economics. However, most participants indicated that they had not studied personal finance in high school or in other college courses.

Lesson 1: Putting the United States National Debt into Perspective

To assess students’ level of understanding of the debt and their ability to make approximations and perform the necessary calculations, students were asked to work in groups to answer the questions that follow. Potential solutions were written on the board and discussed.

- Define the United States national debt.
- Presently, how much is the U.S. national debt?
- If no additional debt were incurred, and the government collected $1 million per day to pay down the debt, approximately how long would it take to pay off the debt?
- Estimate the amount of United States national debt per person?

Results: Lesson 1

Students had a vague idea what the United States National debt was, but their responses seemed to be based primarily on their understanding of the word “debt.” It was unclear whether students could distinguish between the U.S. national debt and the federal budget deficit. Previous research had shown that when given the definition of “a budget deficit,” 22% of students incorrectly responded that the definition was of “the national debt” (Markow & Bagnish, 2005). Despite frequent news articles having mentioned the U.S. national debt, the overwhelming majority of students did not know that as of 2014 the national debt had reached $17.6 trillion. Many students guessed that the national debt was in the millions. After finding the figure online, a few students incorrectly stated the dollar amount due either to having not recalled the names of place values or miscounting digits.

The overwhelming majority of students were unable to estimate the amount of time it would take to pay off the debt if $1 million per day was collected. Students either did not provide an answer or severely underestimated the time to be a few years or a decade. It is important for students to be able to make estimates and judge the reasonableness of answers (NCTM, 2000). However, the strategies students used in this case failed. With the aid of calculators, several students correctly calculated that it would take approximately 48 thousand years to pay off the debt. However, while working, students repeatedly entered the same numbers into their calculators and requested help. It seemed that students were attempting to use their real-world experiences to judge the reasonableness of their answers. For example, because students knew that school loans and loans to purchase a car or home should be paid off in one’s lifetime, they seemed to expect that the U.S. national debt would be paid off in their lifetime as well. Students had difficulty judging the reasonableness of the answer. Difficulties with financial exercises may have arisen, in part, because the real-world numbers resulted in answers that, based on students’ experiences, appeared to be “unreasonable.”

Next, using an estimate of 319 million for the U.S. population, students underestimated the debt per person, which would be approximately $55 thousand. Asked to comment on collecting this amount in taxes from each person, students stated that children, people on fixed low incomes, people who were disabled, homeless, or receiving assistance would not be required to pay taxes. Therefore, the amount of debt per taxpayer might be as high as $110 thousand. After the class discussion, students’ views on the debt and spending had changed, which suggests they had either not considered this issue before or had been unable to put the numbers into perspective.

Lesson 2: Graduated Income Tax

Students were asked to investigate how citizens in the United States are taxed. Students already knew that high earners paid more in taxes, but were not familiar with the term “progressive graduated income tax.” I told the class that as one’s earned income increased, the rate at which one would be taxed increased. Students were given the 2014 tax brackets shown in Table 1. To explain how a progressive graduated income tax works, stu-
Students were told that if a worker’s earned income was less than or equal to $9,075, the worker’s entire earned income would be taxed at 10%. If a worker making $9,075 received an increase of $100, although the total amount of earned income would place the worker in the 15% tax bracket according to Table 1, only the extra $100 would be taxed at the 15% tax rate. Thus, the amount owed in taxes would be $922.50, since 10% ($9,075) + 15% ($100) = $922.50. Therefore, using a very rough approximation, students realized that the total amount paid in taxes would be slightly over 10% of the worker’s entire earned income, since only a small fraction of the worker’s earnings were taxed at the higher rate.

To assess students’ level of comprehension, the class was instructed to work on the following questions:

- How much would a single filer who had a salary of $38,000 pay in taxes and what would be the worker’s after tax salary?
- Find the effective tax rate, which is the average rate at which a person’s salary is taxed?
- What would be the after tax salary of a worker who made $36,000?

### Table 1

**2014 Tax Brackets for Single Filer**

<table>
<thead>
<tr>
<th>Tax rate</th>
<th>Single filers</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0%</td>
<td>Up to $9,075</td>
</tr>
<tr>
<td>15.0%</td>
<td>Over $9,075, but not over $36,900</td>
</tr>
<tr>
<td>25.0%</td>
<td>Over $36,900, but not over $89,350</td>
</tr>
<tr>
<td>28.0%</td>
<td>Over $89,350, but not over $186,350</td>
</tr>
<tr>
<td>33.0%</td>
<td>Over $186,350, but not over $405,100</td>
</tr>
<tr>
<td>35.0%</td>
<td>Over $405,100, but not over $406,750</td>
</tr>
<tr>
<td>39.6%</td>
<td>$406,750 or more</td>
</tr>
</tbody>
</table>

### Results Lesson 2

The overwhelming majority of students miscalculated the taxes owed. Students misunderstood how to interpret Table 1. They incorrectly assumed that a person with a salary of $38,000, who is in the 25% tax bracket, would pay 0.25($38,000) = 9,500. Thus, they assumed the worker’s after tax salary would be $28,500. Students did not understand how a graduated income tax system worked. Having mistakenly calculated the tax, students incorrectly assumed the effective tax rate was 25%. Students found this question puzzling and trivial. Working on the last question, they incorrectly calculated that the worker’s after tax salary would be $30,600, which they found by calculating 36,000 – 0.15(36,000). Students were asked, why a worker making $36,000 would accept a $2,000 raise if it meant that the worker would make less after taxes? Students searched for information online, confirmed the tax rates, and could not answer the question. They did not understand that only the income greater than $36,000 would be taxed at the higher rate. Students also struggled to understand an effective tax rate, but once the financial principles were explained a fair number of students found the computation relatively easy. In this case, students’ misunderstanding related to their reading comprehension and cognitive skills rather than their ability to perform the mathematical calculations.

### Implications and Conclusion

Given the increase in individual fiscal responsibility, students today need to be financially literate. The gaps in financial literacy, which are particularly pronounced among women and minorities, are well documented. It is unclear to what degree low levels of reading comprehension, a lack of familiarity of economic terms and concepts, gaps in elementary mathematics skills, and poor general cognitive skills, have produced low levels of financial literacy.

This paper sheds light on the gaps in knowledge and difficulties college students had by examining their attempts to understand and process financial information. Although this paper examined how a small number of college students worked with a limited number of real-world financial problems, as evidenced by students’ misunderstanding, there are large gaps in their understanding and knowledge of finance, current events, and the ability to make estimates. Given that the vast majority of these students were majoring in Business, these gaps were unexpected. Further research is needed that examines the method by which students approach financial problems to better understand why financial illiteracy is so widespread.

Additionally, at a young age, students need to make important financial decisions including how to pay for college, which cellphone plan to subscribe to, how much auto insurance to carry, and how to fund their future retirement. A lack of financial understanding can have long lasting, severe economic repercussions. Therefore, high schools and colleges should consider integrating financial topics into the current curriculum. Early financial education is invaluable because these essential skills will be important in students’ personal and professional lives outside of school and in their roles as active citizens.
References


