The Theorem of Thales: A Study of the Naming of Theorems in School Geometry Textbooks

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Abstract

An interesting topic for research and reconstruction in the history of mathematics in school textbooks concerns how geometrical theorems were named, and how the name became established within the educational system. In this paper, we examine the name Theorem of Thales, as it emerges at the end of the 19th century, within different cultural, mathematical and educational contexts and how it was attributed to different theorems in European geometry textbooks. In an attempt to explain this phenomenon, we draw upon the concept of didactical reconstruction, a concept which may prove to have wider application to the use of the history of mathematics in school education.

Introduction

Notes concerning history of mathematics have appeared for a long time in school textbooks, at least since 5th century A.D. (textbook of Proclus: *A Commentary on the first book of Euclid's Elements*). The study of these notes may help to clarify our understanding of the evolution of pedagogical views and didactical strategies of textbook authors. As far as we know, there has been as yet no systematic study into how these views and strategies developed[1] . As a contribution to this study, we shall present an analysis of one of the most interesting illustrations of such historical notes in geometry textbooks: how the term *Theorem of Thales* emerged and developed at the end of the 19th century.

Geometrical Education and Textbooks in the 19th Century

National public systems of education emerged during the 19th century, due at least in part to the the impact of the French Revolution. One important feature of these changes was the establishment of *secondary education*, a level of education between elementary education and higher education at universities. One of the results of the pedagogical and didactical demands of this century was that large changes took place in mathematical education, and particularly in the teaching of geometry (Cajori, 1910; Schubring, 1996, 1985). The subject of geometry became established in lower levels of secondary education, and in primary education. Also, the method of teaching geometry underwent radical change in senior grades of secondary education, frequently becoming (as in England, France, and somewhat less in Germany) an exercise in logic (Smith 1900, 303) within the context of introducing students "to the art of syllogism."

Substantial changes also took place in the system of how textbooks were authored and brought into circulation. Many new authors appeared, particularly in secondary education, and the level of circulation of textbooks increased greatly as the number of copies printed rose (Schubring, 1985). Geometry textbooks, in particular, saw growth in the number of translations of important textbooks into several languages. For example, perhaps the most important of these being the geometry textbooks by Legendre and Lacroix (Schubring, 1996, 367), which were translated from the French to several other languages.

Except in England, the content of geometry textbooks (especially of those for the senior grades of secondary education) began to deviate more and more from Euclid's *Elements*, a work that had been the a paradigm of exposition in geometry during previous centuries. As an example for this development, we can consider the significant influence Legendre's *Elements of Geometry* (first edition in 1794) (Schubring, 1996, 366) had both on European textbooks and on those of the United States during the 19th century. In Legendre's book, the study of the circle (3rd Book in *Elements*) preceded that of parallelograms (2nd Book of *Elements*). Thus, the adherents of Legendre modified the Euclidean order of presenting geometry, as they considered the concept of circle simpler and more elementary than that of the parallelogram (Smith, 1900, 230).

Historical References in Textbooks

Towards the end of 19th century, we see *historical references* begin to appear more or less systematically. We use the term *historical references* to designate the elements of a geometry textbook which refer to the history of geometry, and which are written so that omitting them from the textbook would not be detrimental to its conceptual geometrical content. During the first stage of their appearance in textbooks, historical references were never an organic part of the main part of text. Instead, they are generally contained either in the book's general introduction, or in the introductions of chapters, or at a chapter's close. If they do appear within the text, it is as commentary in brackets and/or small point, or even in footnotes (which are usually in smaller point)). As a rule, these historical references refer to the work of mathematicians of antiquity, by *naming* of theorems the latter are held to have proven, or not to have proven. Sometimes, such references contain a brief summary of the evolution of geometry, or a note relating to a specific geometrical subject, and to related dates. Such references were not altogether novel in geometry textbooks, they were present in older textbooks as well. However, it was not a recognized technique, but one used by only a few authors. For example, there are summaries of geometry's evolution in textbooks dating from the 17th century, as in Beaulieu ,1676; Leyboyrn, 1690; Le Clerc, 1690 (Kokomoor, 1928, 101). The name "Lunes of Hippocrates" appears in a textbook of the 17th century[2] as well.

We believe that the increased use of historical references in geometry textbooks at the end of 19th century is related to some extent to the substantial advances made in the historiography of mathematics after 1870, mainly in England, France, Germany and Italy, in connection with the rise of interest in historical studies in general (Allman 1877, 160- 161). This led some textbook authors and teachers of mathematics to try to "use" history in the teaching of mathematics, in a more "systematic" way than before (Dauben 1999, 110, quoting G. Eneström). The introduction, in particular, of references to *Ancient Greece* can be explained by the rising general interest, in Ancient Greece and Greek mathematical works[3] in the course of the 19th century. Some of these references simply consist of attaching a *name to theorems*. One of these peculiar historical references introduces the name *Theorem of Euclid* (only in German textbooks), and another, which we shall study below, introduces the name *Theorem of Thales*.

Geometrical Achievements Attributed to Thales by Ancient Sources

In Ancient Greek sources, we find: i) five *major references* to geometric achievements of Thales and ii) some other references concerning his calculation of the height of Egyptian pyramids in Plutarch, Hieronymus the Rhodian, and Pliny[4]. Four of the five major references are found in Proclus. They attribute to Thales the following specific theorems: the circle is bisected by its diameter, the angles at the base of an isosceles triangle are equal, the opposite angles are equal and two triangles are equal when they have one side and two adjacent angles equal (Thomas 2002, 164-167). The other major reference is found in Diogenes Laertius's biography of Thales, quoting Pamphila's testimony that Thales "was the first to inscribe in a circle a right-angled triangle" (Thomas 2002, 166-169). Among historians of mathematics, we find diverse views about the correctness of attributing the above theorems to Thales[5] .

In the case of the work (generally ascribed to Thales) which involved measuring the height of pyramids, historians of the 19th century usually concluded that Thales knew some basic principles of similarity (similar triangles), but they rarely attributed a specific theorem to him. We know of only one work in the history of Mathematics which explicitly mentions two theorems relevant to this type of measurement (Ball 1908, 14-16), and these are theorems of Euclid's *Elements:* "If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally" (VI, 2) and: "In equiangular triangles, the sides about the equal angles are proportional" (VI, 4) (Heath 1926, 194-5, 200-1).

Three historians mention the name *Theorem of Thales*, but two of these only mention it to reject it (Loria 1914, 22) (Tannery 1930, 67), and the third, G. Eneström, voices serious objections to the appropriateness of the name (Enriques 1911, 57). The case of D.E. Smith is different. He was active both as a recognized scholar of the history of mathematics and as a textbook author. It is notable, however, that he never used the name *Theorem of Thales* in his texts, despite the fact that his textbooks contain historical references to Thales (Wentworth-Smith, c1913, 32).

How the Name *Theorem of Thales* Emerged and Became Established

Early attributions of various theorems to Thales

Many years before the name *Theorem of Thales* emerged, historical references appear in geometry textbooks attributing various geometrical achievements to Thales. For example, both the Modern Greek translation of Tacquet's textbook (Voulgaris, 1805, 25) and its original in Latin (Tacquet, 1722, 20) attribute to Thales the calculation of the distance to the inaccessible points by applying the theorem which states that two triangles are equal when one of their sides and two adjacent angles are equal. Also, Benjamin of Lesbos credited Thales with the theorem about the angle inscribed in a semicircle, as well as the theorem that opposite angles are equal (Benjamin of Lesbos, 1820, 90, 21). The same author also mentions that Thales calculated the height of Egyptian pyramids by using the proportionality of the sides of similar triangles (Benjamin of Lesbos, 1820, 6).

French textbooks

The name *Theorem of Thales* first appears in a few French textbooks before the end of 19th century, as early as 1882, (Rouché and Comberousse, 1883, cited in Plane, 1995, 79). The name is attributed to the (general) *theorem of proportional line segments or theorem of proportional lines*: "Des droites parallèles déterminent sur des sécantes quelconques des segments proportionnels" (Parallel lines determine proportional segments on any lines which they cut). However, the very same name is attributed to at least two special cases of the general theorem, as e.g.: "Toute parallèle à l'un des côtes d'un triangle partage les deux autres côtes en parties proportionelles" (All lines parallel to one of the the sides of a triangle cut the other two sides proportionally) (Combette, 1882, cited in Plane 1995, 79) and "Dans le triangle, l'égalité des angles entraîne la proportionnalité des côtes" (two triangles with equal angles will have proportional sides) (Rouché and Comberousse, 1883, cited in Plane 1995, 79). By the 1920's, the name Theorem of Thales was well-established in French geometry textbooks, and mentioned in the

1925 French curriculum as well (Bkouche, 1995, 9). It also appears in textbooks of Descriptive Geometry (Cholet- Mineur, 1907- 1908, 315).

Italian textbooks

The theorem of proportional line segments also bears the name of *Theorem of Thales* in Italian geometry textbooks (Faifofer, 1890, 262), at least since 1885 (Enrico 1885, 34). It also appears in Italian textbooks on analytic geometry (Enrico, 1885, 34) and analytic-projective geometry (Burali- Forti, 1912, 92).

German textbooks

The name *Theorem of Thales* is also used in some German textbooks written at the end of 19th century, at least since 1894, but here, it is attributed to a completely different theorem: "Der Peripheriewinkel im Halbkreise ist 90° "(The angle inscribed in a semicircle is a right angle) (Schwering and Krimphoff, 1894, 53). This name is used for the same theorem (possibly with some variations in the theorem formulation), in German textbooks during the first decades of the 20th century. It also shows up in a German encyclopedia of mathematics (Weber-Wellstein- Jacobsthal, 1905, 232).

English and US textbooks

The name *Theorem of Thales* neither appears in American textbooks, nor in those published in England. In the US, however, we do have references to Thales concerning both his geometrical achievements and the measurement methods attributed to him[6]. Several of these *historical references* are due to D.E. Smith (Wentworth/Smith, c1913, 454, 466). In English textbooks, historical references were generally rare during the 19th century.

Other European textbooks

As a consequence of the cultural impact of France and Germany on several other European countries, the name *Theorem of Thales* also shows up (with different meanings) in those countries' textbooks. Thus, the name *Theorem of Thales* appears in Spanish (Deruaz-Kogej, 1995, 2390, Belgian (Cambier 1916, 142), and Russian textbooks (Kastanis, 1986, 3) in the same sense as it is used in French and Italian textbooks[7]. The same name, Theorem of Thales, is employed in Austrian, Hungarian (Howson 1991, 21) and Czech textbooks (Pomylaková 1993, 62) however, but with the meaning attributed to it in German textbooks, rather than the French and Italian meaning. Modern Greek textbooks present an exceptional case: at first they used the name of the Thales theorem in a sense adhering to that of German textbooks (Hadjidakis, 1904, 60), and later they switched to the sense given to it by French textbooks (Nikolaou, 1927, 128); (Barbastathis 1940, 136).

The Naming of Theorems in the Context of Mathematical Education of the 19th Century

The study of how theorems were *named* in different countries, some of the variation having been shown above, reflects the different (cultural) conditions prevalent in the mathematical education of the countries concerned. An important example is provided by France with its time-honored anti-Euclidean outlook (Schubring, 1996, 377; Cajori, 1910, 182). This viewpoint resulted in a tradition which caused the French order of geometry subject-matter to deviate from that of Euclid. For example, in Euclid, the theorem of the square of the hypotenuse (I, 47) precedes the theory of proportions (5th book of *Elements*) just as well as the theorem corresponding to that of proportional lines (VI, 2). In French textbooks, this order had been reversed since P. Ramus's era. This reversal was not followed by German textbooks until the beginning of 20th century[8]. Italian textbooks (after 1866) retained the Euclidean order presenting geometry subjects because, at that time, Euclid's *Elements* had been adopted as the official textbook in Italian schools (Schubring, 1996, 377-378; Cajori, 1910, 191).

In the meantime, the theorem of proportional lines had risen to a prominent position in French textbooks, because of the new developments in (academic) mathematical research in geometry. One of these developments which following the the works of G. Desargues, B. Pascal, La Hire (1685), Carnot (Coolidge, 1934, 219-220), was due to the work of J.V. Poncelet (of 1813, published in 1822) marked the beginning of *Projective Geometry and Affine Geometry*. A notion maintaining a key position in projective geometry is *harmonic separation* (Coolidge, 1934, 220), which refers to an invariant ratio of particular line segments and thus relates to the theorem of proportional lines. This theorem also relates to the new subject of affine geometry because the theorem of proportional lines implies preservation of ratios between collinear line segments which is a key notion in affine geometry. These new "structural" mathematical developments had in some sense been *institutionalized* in the teaching of geometry by the end of 19th century[9]. Simultaneously, historical interest in Thales as a mathematician grew in the 19th century, this interest possibly being one of the reasons why the name *Theorem of Thales* became the generally accepted designation for the above theorem[10].

The same situation is all the more true for Italian textbooks. Although Italian textbooks maintained the Euclidean order of presenting theorems, Italian authors, particularly L. Cremona, adopted certain elements from projective geometry (Cajori, 1910, 191). Some Italian authors, like R. de Paolis, 1884 (Candido, 1899, 204) also tried to blend elements from plane geometry and the geometry of solids, again by drawing on ideas taken from projective geometry. This helped create a "preference" in Italian authors for the theorem concerning proportional line segments, which they attributed to Thales.

The case of German textbooks is different, since the Germans did not change the Euclidean presentation. Note for example that they showed strong preference for the theorem concerning the square of the hypotenuse, the so-called *Pythagorean Theorem*[11]. This preference exemplifies a school geometry which was "more aligned" to Euclid, and thus lacked any conceptual association with projective or affine geometry. (However, this is not to suggest that academic research in

Germany did not contribute to the development of projective geometry after 1830).

It would appear that *names* are assigned to theorems considered essential and significant for school geometry[12]. Thus, French textbooks chose to ascribe the name Theorem of Thales to a theorem which was essential for the modern view of geometry (projective, affine), while German textbooks chose to ascribe the name to a theorem important for "classic" Euclidean geometry, but containing a large number of modern elements (algebraic calculations). Modern European textbooks in other countries were far removed from Euclidean geometry, and saw the necessity of stressing the importance of theorems essential to new developments in geometry.

Towards a Didactical Explanation of the Name

Our study pertains to those contents of school textbooks which do not refer directly to mathematical concepts, but to the "historical origin" of those concepts. Textbook authors were always looking for ways to emphasize the importance of certain theorems in the curricula. *Naming* a theorem is a symbolic act which goes far beyond presenting a simple historical landmark. After having introduced the history of a concept or theorem which the textbook authors had emphasized in their text, they tried "to profit" from history. Naming the theorem after an undisputed, time-honored authority to which it could be historically traced further authenticates it and establishes its importance.

In the first phase of this process, we find some simple historical references to Thales, without naming of theorems after him. A French textbook of 1866 (Rouché and Comberousse, 1866, v) for example, attributes a theory of similar triangles to Thales, while a German textbook of 1875 (Kruse, 1875, 18, 64) attributes two theorems (opposite angles are equal; in a semicircle, the angle is a right angle) to the same Thales. In a later stage, the historical references to Thales evolve into *naming* of the corresponding theorems after him, and into using of his name not in footnotes or asides, but within the main body of text. Establishing the name *Theorem of Thales* in this later stage served a didactical need by using the history of geometry to underpin the particular didactical-ideological interpretation the textbook authors favored. We are thus led to speak, in analogy to *didactical transposition* (Chevallard and Joshua, 1982) about a *didactical reconstruction* of History of geometry, i.e. a reconstruction of history of geometry for didactical requirements.

The authors of school textbooks are as a rule not historians of mathematics;. Rather, they are teachers who evidently desire to "profit" didactically from the history of geometry. They seem to know about Thales from a historical source, and try to establish a direct link between Thales and a theorem already present in school geometry. There is no mention of sources from antiquity at all upon introducing attributions of theorems in textbooks of school geometry, particularly not in connection with the name *Theorem of Thales*. What we find are only different *choices* from historical sources among these theorems ascribed to Thales, with no discussion or historical argumentation at all. Historians of mathematics proceed in a totally different way. For example, they argue at length whether the ancient sources about Thales are valid, or not, trying to see if Thales indeed gave proofs, and if he employed some general method.

We consider that the concept of *didactical reconstruction* offers a key to understanding how textbook authors used the history of mathematics. They focused on its didactical advantages, treating history as one educational tool among others. Here, we have shown that to emphasize a theorem's value, they attributed it to a famous mathematician of antiquity (Thales). In other cases, they selected topics or subjects from the history of mathematics according to their own national and cultural tradition. We believe that there is more to investigate and learn in mathematics education by examining historical references in school textbooks. The name *Theorem of Thales* is only a particular, though telling, example in this direction.

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Notes

1. M. Gebhardt's work, *Die Geschichte der Mathematik im mathematischen Unterrichte*, 1912, remains an exception, for a description of the book see (Furinghetti, 2001, 1).
2. The name *Lunes de Hippocrate de Scio* appears in the textbook *Elemens de Geometrie* of the Jesuit Pardies (1676), as well as in Tacquet's textbook (1745) (Lietzmann, 1912, 35). The name appears also in the Modern Greek translation of the Tacquet textbook as *conjugate lunes of Hippocrates of Chios* (Boulgaris, 1805, 296, 302), This translation draws on the 1710 edition of Tacquet's textbook (Karas, 1993, 70).
3. There were many editions of ancient Greek mathematical works at the end of 19th century, for examples see (Allman, 1877, 161).
4. There is an exhaustive presentation of the literature about Thales and geometry in (Tezas, 1990, 61-103).
5. We refer to three of the most important of these historians, (Cantor 1907, 134-147; Heath 1921, 128-137; Tannery 1887, 81-94). At this point, we should like to make a distinction between the research works in the history of mathematics, and the works addressed to a wider audience (popular works). In any case, we have not found a book on history of mathematics written only for the wider audience in the second half of the 19th century.
6. As examples, we may mention the calculation the height of Pyramids, and measuring inaccessible points (Betz and Webb, c1912, 281, 68). In another textbook, among other

achievements, the theorem that every diameter bisects a circle is attributed to Thales (Fletcher, 1911, 496).

7. In Russian textbooks, we read the following theorem as *Theorem of Thales*: if two transversals are given and three or more parallel lines intercept congruent segments on one of them, then they also intercept congruent segments on the other one. In addition, the theorem of French and Italian textbooks was sometimes named *Generalized Thales' Theorem* (information by professor G.Schubring). In later Greek education, the theorem about transversals had the name *Lemma of Thales* (information by professor T. Patronis).

8. This change of order had an impact on the proof of the corresponding theorems (I, 47 and VI, 2). We observe that the complete proof of the theorem VI, 2 and the analogous theorems of section 5.2 requires the theory of proportion (book V), or, in general, some theory of irrational numbers (limits or Dedekind cuts). The historical stages in the development of the proof of the theorem of proportional lines from Euclid to Dedekind may be found in Bkouche, 1995, but textbooks contain a great variety of incomplete proofs, which have their own interest and their own history. We note that textbook authors do not relate the name of a theorem to its *proof*, otherwise they would not attribute to Thales (a beginner of geometry) a theorem which requires so difficult a proof.

9. According to Patronis (2002, 68), institutionalization is, first and foremost, a symbolic act of showing what is important and respectable within human society, or within a context of social activity. In this sense, institutionalization in the context of (mathematical) education uses (names of) historical figures as Thales, Pythagoras, and sometimes also Euclid to assign a status to the subject taught.

10. More generally, there was a growing historical interest in early Greek mathematics fostered by C. A. Bretschneider's *Die Geometrie und die Geometer vor Euklides*, 1870, see (Allman, 1877, 161).

11. There are at least four books in German which study this theorem: J. Hoffmann (1819), J. Wipper (1880), H.A. Naber (1908), of course, Lietzmann, 1912. Lietzmann refers to Hoffmann, Wipper and Naber at the page 70).

12. This is also the case for the name *Pythagorean Theorem*, as well as for the name of *Theorem of Hippocrates of Chios* in Modern Greek textbooks of Geometry (Patsopoulos, 2003, 577).

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