Exams in Algebra in Russia: Toward a History of High Stakes Testing

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Abstract

This article analyzes the history of Russian graduation exams in mathematics from the end of the nineteenth century to the middle of the twentieth. The author reconstructs this history by relying both on official documents and on the memoirs of former students, contemporary journalism, and methodological literature. The analysis brings to light problems that were associated with the writing and administering of exams in the past, along with the historical debates surrounding these problems—debates that continue to have relevance today.

Introduction

Few topics in education are likely to attract more public attention than high-stakes testing and school accountability. An Internet search for these keywords yields literally hundreds of thousands links to stories in the mass media. Nor is there any lack of scholarly publications that analyze one or another aspect of such testing (e.g. Carnoy et al., 2003; Heubert and Hauser, 1999). Publications devoted to the comparative analysis of the way in which exams, including exams in mathematics, are conducted in different countries also exist, although in significantly smaller numbers (e.g. Dossey, 1996). Less attention is usually paid to the historical analysis of the ways in which such testing was carried out in the past (e.g. Madaus et al., 2003). The historical analysis of exams, however, appears to us to be of particular interest, since it makes it possible simultaneously to analyze what Schubring (1988) has called “administrative history”—the trajectory of changes in official directives and programs—and what he has called “real history”—that which actually took place in the schools. From this perspective, the purely mathematical problems offered on exams can be interpreted as manifestations of specific social views, and the changes in their types and structures can be seen as expressions of—or at least wishes for—social changes.
This article will deal with Russian written exams over a long period of time—from the 1890s to the mid-1950s, when the Soviet system of exams finally acquired the form that largely remains in place in Russia to this day. It should be noted at once that both the size of the present study, and occasionally the lack of data, often compel the author to limit himself to schematic descriptions, omitting many details (see also Karp, 1998). The author’s principal focus will be on exams in pre-Revolutionary gymnasia and in Soviet high schools—even though other types of secondary educational institutions did exist; on graduation exams—even though students also went through exams in advancing from grade to grade; and on exams in algebra—even though there were also graduation exams in other mathematical subjects (for example, geometry). The author will also often have to ignore certain geographical or even chronological distinctions. For example, even during the period of Stalinist hyper-centralization, certain differences inevitably arose between the ways in which exams were administered in the capital and in provincial towns. In addition, neither is it clear that the exams from any two years in succession were ever perfectly identical. It should be emphasized that a vast quantity of materials pertaining to this subject matter remains unanalyzed.

On the Historical Period under Investigation

The period that this article will deal with is full of dramatic events in Russian history. The comparative liberalization of Russia’s autocratic regime that was undertaken by Tsar Alexander II was cut short by his assassination in 1881 and supplanted by the traditional-conservative policies of his son Alexander III. The next and last Russian monarch, Tsar Nicholas II, tried to follow the same conservative course as his predecessor. However, under the pressure of growing popular resistance—above all, the Revolution of 1905–07—he was forced to accept certain reforms. Yet these reforms proved highly inadequate. The economic and political situation in the country became critical during the course of the First World War, and led to the Revolution of February 1917, which put an end to autocratic rule. In October 1917, however, control of the government passed to the Bolsheviks, who gradually built a new state, the USSR. In the history of the USSR, too, periods of comparative—although limited—liberalism alternated with periods of harsher autocratic rule. The death of Stalin in 1953 led to the well-known liberalization carried out by his successor, Khrushchev, but even during Stalin’s long reign the situation in the country was not always the same; the years following WWII, for example, were marked by a increased “tightening of the screws,” while the very end of the 1930s witnessed a certain (even if very limited) easing after a period of mass terror.

The education system reflected what went on in the political life of the country. In 1905–07, we see attempts at radical reforms in education (Tolstoy, 2002), while in 1917, following the October Revolution, the old system by and large simply falls apart. In 1946-53, by contrast, we see attempts as far as possible to reinstate the pre-Revolutionary system, which now has come to be seen as a model of conservative stability.
Sources
The surviving sources on the history of mathematics exams may be divided into three groups. The first consists of surviving official documents about exams, including reports about their administration and results. The second is comprised of methodological articles, speeches, manuals, and problem books, which analyzed exam problems, their various solutions, how these solutions ought to be written on exams, or how students ought to prepare for exams. Finally, the third and no less important group of sources is made up of materials reflecting public discussions about the administration of exams in general and mathematics exams in particular; and detailing students’ and teachers’ personal experiences. This type of literature is extremely varied, ranging from humorous stories to minutes of city council meetings, and includes a large number of memoirs and magazine articles.

The need to study how student performance on exams was assessed in the past, and to analyze documents related to this topic, has long been noted (Ganelin, 1954). It may be argued, however, that the informal sources mentioned above constitute evidence that is no less important than the official documents: the real picture of exams such as they were can be reconstructed only by taking into account their reception by “ordinary” people and not just professional mathematics educators.

Some General Facts about Exams
Before the Revolution of 1917, without passing a graduation exam students could not obtain a diploma, which was required to enter a university. There were graduation exams in virtually all subjects (written in some cases, oral in others). However, the number of students who took graduation exams was comparatively small. In St. Petersburg in 1896, for example, a total of 612 people took exams in mathematics: 458 in government gymnasia, 34 in private gymnasia, 64 in gymnasium branches of educational institutions affiliated with Lutheran churches, and 56 “unaffiliated students”1 (“Iz otchetov”, 1897, p. 444). The overwhelming majority of young people, even in St. Petersburg, did not attend gymnasia (or other types of secondary schools, such as the so-called real schools modeled on German Realschule, whose graduates were not allowed to enter St. Petersburg University, but could enroll in technical institutes). Nor did everyone who entered a gymnasium stay to complete it. The exams themselves functioned as a kind of obstacle that prevented students from continuing their education, and grading was usually quite strict. This particularly concerned “unaffiliated students,” who wished to obtain diplomas without attending classes, simply by passing exams. Thus, among the unaffiliated students taking exams in St. Petersburg in 1907, 48.8% failed to obtain a diploma; by comparison, in the country as a whole only 3.9% of the graduates of government gymnasia failed to obtain a diploma (“Nashi uchebnye”, 1908, p. 47).
It is not surprising that public opinion about exams was often quite negative—they were seen as part of the system of bureaucratic training that placed restrictions on people’s freedom. A characteristic discussion took place in the revolutionary year of 1907 in the Voronezh city council. City council member Andrey Kiselev, one of Russia’s best known mathematics educators and the author of numerous textbooks, spoke out in defense of exams, describing at length their various virtues (Karp, 2002). Nonetheless, the city council voted to uphold the proposal that exams be discontinued. It should be noted, however, that public attitudes toward graduation exams were usually more favorable than toward grade-advancement exams (Ganelin, 1954, p. 294).

After 1917, all exams were, naturally, cancelled (Polozhenie, 1918, p. 110). For a time, it became commonplace to view exams as meaningless and harmful. As the Pedagogical Encyclopedia (1929) pointed out: “It is impossible to form an accurate impression about a student’s knowledge and intellectual development through exams” (p. 581). Following the August 25, 1932 decree of the Central Committee of the Communist Party (Direktivy, 1947, p. 164), however, it turned out that this was not only possible, but in fact necessary. It was conceded that post-Revolutionary pedagogy had been misguided, and after a certain period of confusion (during which, for example, it was considered important to emphasize the differences between the new exams and the old, pre-Revolutionary ones, which were characterized by their scholastic formalism—“O predvaritel’yakh,” 1933, p. 3), graduation exams, including graduation exams in mathematics, once again acquired their former status. But now they were administered to hundreds of thousands and even millions of children, since the number of young people who completed secondary education rose substantially and steadily. It is important to bear in mind, however, that along with graduation exams, separate entrance exams at colleges and universities were established. Thus, a high school diploma was necessary for entering an institution of higher learning, but not sufficient; students were also required to pass other exams, which varied from one educational institution to another.

The Goals of Exams and Their Role in Ensuring Quality of Education

As has already been noted, why exams were needed at all, and why graduation exams were needed in particular, was the subject of considerable debate. The 1891 Rules for Testing Gymnasium and Pre-gymnasium Students declare: “testing is aimed at determining whether those subjected to it have attained to a sufficiently advanced stage of intellectual development and are fit to pursue to further scientific education profitably and successfully” (“Pravila,” 1891, p. 221). Exams had the following essential goals, as formulated both in the press and in official documents:

- to determine each student’s level of preparation;
- to motivate students to carry out a systematic review of a course and to assume greater responsibility for their own education;
• to monitor the quality of education in educational institutions;
• to verify that established educational curricula were being followed in all details;
• to collect objective data about the results of education.

These goals were discussed, in one way or another, both before the Revolution of 1917 (e.g. Kapterev, 1892; Vysotsky, 1894) and after (e.g. О проведенни, 1934). Exams played a particularly important role as a means of monitoring the quality of education in schools.

According to the 1891 Rules for Testing Gymnasium and Pre-Gymnasium Students, “the transcripts and records, lists of names and grades, and the written exams of all those undergoing testing are subjected to a thorough examination at the district administrative office and assessed in a report on the progress of testing in the gymnasia of the district” (“Pravila,” 1891, p. 243). In practice, this meant that some professor from a local university was required to review and re-grade all of the exams from the district, and to arrive at a verdict regarding the state of education and the work of the teachers in local gymnasia. Such reports were usually published, read out loud at assemblies, and otherwise disseminated, so that, as one of the authors discussing the practice of administering exams noted, “a teacher was held up for all the world to see” (Istrin, 1908, p. 57). The professor, in such cases, would often turn out to be a far more exigent grader than the teachers. For example, the following grades were given to graduates from the gymnasia of Kiev on their algebra exams by their teachers and by Prof. Suslov (Suslov, 1909). The highest grade is “5”; a “2” is a failing grade.

Table 1.

<table>
<thead>
<tr>
<th>Grades</th>
<th>“5”</th>
<th>“4”</th>
<th>“3”</th>
<th>“2”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers’ grades</td>
<td>53 students or 15%</td>
<td>78 students or 22%</td>
<td>199 students or 54%</td>
<td>34 students or 9%</td>
</tr>
<tr>
<td>Professor’s grades</td>
<td>25 students or 7%</td>
<td>64 students or 18%</td>
<td>217 students or 59%</td>
<td>58 students or 16%</td>
</tr>
</tbody>
</table>

In the post-Revolutionary period—or more precisely, starting in the mid-1930s—the practice of “independent” inspections by individuals outside the schools was revived in one form or another, although it never became as thorough and exhaustive as it had been before the Revolution. For example, D. Goncharov (1938) writes about re-checking 328 algebra exams taken by students at 14 different schools in Odessa (in a district where exams had been administered to graduates from 117 classes in all). Among other things, he notes that teachers give significantly inflated grades, in part because they themselves make or overlook mistakes. Selective inspections of this kind continued to be carried out at other times and places. For example, in 1948, a special manual for analyzing graduation and grade-advancement exams was published in Leningrad.
(Matematika, 1948). Clearly apprehensive of being accused of besmirching Soviet reality, the well-known mathematics educator Ivan Depman could nevertheless not refrain from writing in this manual about the practice of systematic grade inflation by teachers: “although it would be wrong to consider these cases to be in any way characteristic of Leningrad schools, it is nevertheless impossible not to point to them as facts that are by no means outside the limits of conceivable possibility” (p. 43).

Procedures for Developing and Administering Exams

The procedures for administering exams prior to the Revolution were governed as a whole by the aforementioned Rules of 1891. The Rules left much within the discretion of the educational district. In principle, exam problems were sent to the schools from the district office. In practice, however, this could mean any number of things: the district office might simply stamp its approval on problems that had been prepared at a gymnasium; or it might select from among different problems that had been submitted by a gymnasium; or it might prepare the problems itself, without any input from the gymnasium. Other procedures were also possible, for example, exams might be sent directly from the Ministry of Education (“O poriadke,” 1912).

In contrast with all this variety, there was unanimous insistence on the strictest order during the administration of the exams. The Rules of 1891 required that each student taking an exam be seated at a separate desk, standing apart from the others, and teachers were advised not to allow students to leave the examination room for the duration of the exam (two and a half hours). The students were very closely monitored. In his report on the exam in algebra at the First Gymnasium of Baku, Tirpotin (1912) writes that “every observer was assigned a group of 3-4 persons by the director” (p. 13).

These facts are corroborated by evidence from former students: “Graduation exams were administered under conditions of extraordinary discipline,” writes the great theater director and actor Konstantin Stanislavsky (1962, p. 64). There are also memoirs that describe the techniques employed to prevent cheating on exams: “They turned our pockets inside out, examined our handkerchiefs, watches, shirtsleeves, and even made us take off our boots” (Ganelin, 1954, pp. 287-288).

In Soviet times, examination procedures went through substantial transformations. If the Instructions of 1933 require that testing be conducted in an ordinary setting (“Instruktsiya,” 1933, p. 14), then somewhat later documents from the very same year already criticize “attempts to transform tests into ordinary class sessions” (“O predvaritel’nykh,” 1933, p. 3). The 1936 Instructions for Administering Exams explicitly specify how many and what kinds of problems exams should have, and assign the task of writing them to the district offices of the Ministry of Education (O provedenii, 1936). Once again, leaving the classroom during the exam becomes prohibited, as does writing on paper that does not have an official stamp on it, etc. (Berezanskaya and Margulis, 1938).
Central control over exam content and administration reached unprecedented levels after WWII. Certain formulations in the Instructions (*Ob ekzamenakh.*, 1945) are clearly borrowed from the pre-Revolutionary Rules: “Students taking exams must be monitored by members of the examination committee at all times” (p. 6). Yet a fundamental new feature appears as well: the problems for the now five-hour-long exam are from henceforth written only by the Ministry of Education in Moscow, and from there distributed around the entire country.

**Breaking the Rules**

The picture of exams in pre-Revolutionary schools that emerges from memoirs of former students, however, is quite different from what is suggested by the official instructions. Alexandra Brushtein’s young female protagonists (1985, pp. 611-612) find out from the teacher, in advance, the problems that will be on their exams. Yurii Annenkov (1991, pp. 53-54) was able successfully to graduate from his gymnasium only because the district supervisor administering his exam in Latin took a liking to him and gave instructions to change his grade in mathematics. Stanislavsky’s (1962, p. 65) classmates are experts at cheating, and so on. None of this was a secret for anyone. An author hiding under the pseudonym Pravdin (the Russian word “pravda” means “truth”) explains at the length that children cheat, and that their teachers condone it, and that there “even exist educational institutions whose directors openly demand of the teachers that they tell their students about everything that will be on their exams and, on top of this, that they help them complete them in every detail”—and that not even in the educational district offices is everything entirely wholesome (1900, p. 64).

Cheating techniques are described in detail in the article, “Kak u nas ‘delaiutsia’ ekzameny” (“How Our Exams Are ‘Made’”) (1913). These include teachers solving exam problems beforehand, in front of the entire class; monitors “reading newspapers” during exams and thus not noticing that students are passing answers to one another; passing exam problems to specially invited experts—while the exam was taking place—and receiving them back with the solutions; and much else besides. The Ministry was quite aware of all of this (“O poriadke,” 1912) and, as one countermeasure, proposed preparing several different exams from which the director of the gymnasium could select one at the last minute.

How such measures were supposed to help to prevent students from simply copying answers from one another (even in those cases when the director was impeccably honest) is not entirely clear. On the other hand, it is obvious that the use of a single exam across the entire country in the USSR after WWII created new possibility for cheating. For example, from Moscow or Leningrad one could call Vladivostok, where due to the time difference exams would end long before they would begin in the European part of the country, and find out the problems that were on the exam (Rozovskaya, 2005). Moreover, none of the old techniques for cheating described by the pre-Revolutionary author disappeared. Depman (*Matematika*, 1948) is mathematically precise about the matter: “The possibility of cheating has not been eliminated in all schools” (p. 43).
Both Pravdin before the Revolution and Depman after it remained quite skeptical about the very possibility of reaching any conclusions about the quality of education on the basis of exams that were administered in such a lax fashion. “Is it possible, on the basis of students’ performance on exams in mathematics, to form an approximately accurate judgment about how the subject is taught?” Pravdin asks—and he goes on: “My reply is, decidedly, yes—but only if the exams are properly administered” (which, as he himself demonstrates, is never the case). Zimmerman (1913, p. 4) draws even more general conclusions, pointing out that educators have failed to achieve standardized requirements in the administration of exams, in the behavior of the teachers administering the exams, and even in the difficulty of exam problems.

**On the Subject Matter and Structure of Exam Problems**

The 1891 Rules called for only one problem on the graduation exam in algebra. But this was a problem of a special kind. Let us examine one example of such a problem, from an exam given at the Vvedenskoy Gymnasium in St. Petersburg in 1910 (Karp, 1998):

A certain school has over 100 but fewer than 200 students. If the number of students is divided by the second term of the geometric progression (1) which has as many terms as twice the solution of the equation

\[2 \left(1 + \frac{9}{x}\right) + 3 \sqrt[3]{\frac{x + 9}{x}} = 14,\]

(2) whose first three terms add up to 35, and (3)

whose remaining terms add up to 280, then the remainder will be equal to 5. But if the number of students is divided by the solution of the equation

\[\sqrt[3]{x - 9} - \sqrt[3]{3 - y} = 0,\]

then the remainder will be equal to the solution of the equation \(A(m, 4) = 12A(m, 2)\) (where \(A(n, k)\) is the number of permutations of \(n\) elements taken \(k\) at a time). How many students are there in this school?

Such a problem contains several sub-problems. It requires students to solve irrational and exponential equations; to address issues connected with combinatorics; to work with progressions; and to solve equations in integers. This selection of topics is quite typical on pre-Revolutionary exams. Contemporary problem books for exam preparation (for example, Bychkov, 1909; Ipatov, 1910; Isachkin, 1915) prepared students precisely for such subjects. Also common were logarithmic equations, quadratic equations, problems on the binomial theorem, word problems, and continued fractions. Over the years, there developed something like unofficial lists of topics (more specific than the official curriculum) that students could expect to encounter on exams. The literature on exams, incidentally, contains proposals to make such lists official (Ya. G., 1890). But even without this official designation, practice exams functioned as a kind of unofficial guide to what was important in the school mathematics curriculum.
What strikes the modern reader as remarkable, however, is not the subject matter of the problems, but their structure. As illustrated by the algebra problem cited above problems on the exams were multi-step with several interdependent parts. Yet it was precisely problems with such a structure (we will call them “composite problems”) that predominated for an extended period of time. Suslov (1909), who has already been quoted above, deliberately criticized a gymnasium (the only such gymnasium in its district) where students were given three separate problems on their exams, instead of one composite problem: “Such a substitution makes matters more convenient [for the students], since a mistake made in the solution of any one of the problems has no influence on the solution of the other problems” (p. 19). It also seemed to many educators to be unfair toward students at other schools; and even more importantly, wrong in principle, since solving individual problems that were based on stock patterns was considered too easy and not an adequate means of testing students’ intellectual development (e.g. see Trudy, 1900, pp. 51-52).

Naturally, other views were also expressed, especially in the years immediately preceding the Revolution. For example, in his speech before the first Russian National Convention of Mathematics Teachers, Markovich (1913, p. 179) sharply criticized the “fantastically complicated combinations of mathematical problems, which [students will] never encounter either in any practical application or at any further stage of theoretical mathematical learning.” In his view, the ability to solve such problems proved nothing beyond a certain degree of precision and orderliness.

Official directives from the Ministry also contained criticism of problems that were “simplistic in content, but made extremely complicated through supplementary computations” (“Po voprosu,” 1912, p. 144). As a consequence, the number of composite problems on exams was reduced. Separate, self-contained problems began to take their place. For example, the following questions appeared on the exam given at the St. Petersburg Historical-Philological Gymnasium in 1913 (Karp, 1998):

1. Find the sixth digit after the decimal point in the decimal notation of the greater root of the equation \( 2x^2 - 4x + 1 = 0 \).

2. Find the geometric progression \((a_n)\), given that \( a_1 + a_2 + a_3 = 13 \),
\[
\frac{a_1 + a_2}{a_2 + a_3} = \frac{1}{3}
\]

3. Solve the following equation in positive integers \( 8x - 10y = 18 \)
At times, writers also expressed the view that exam problems should be "connected to life," for example, by being associated with physics (Trudy, 1900, p. 42). But apart from typically artificial word problems, no problems of this kind appeared on exams.

It is interesting to note that when graduation exams reappeared in the mid-1930s—now written at the district offices of the Ministry of Education—composite problems once again showed up on exams. They did not appear everywhere, of course, but they were nonetheless present in many districts (Shreider, 1937; Berezanskaya and Margulis, 1938). Now they were criticized very severely as providing no basis for assessing student knowledge, since students might be unable to complete the main problem due to a mistake made in one of its components. Nevertheless, the following problem was given in Kalinin in 1937:

The lengths of the sides of a right triangle form an arithmetic progression, while its area is 10 times greater than a root of the equation $\frac{\sqrt{x+1} + \sqrt{x-6}}{\sqrt{x+1} - \sqrt{x-6}} = 7$. Find the lengths of the sides of the triangle (Berezanskaya and Margulis, 1938, p. 66).

The selection of topics for exam problems also turned out to be quite conservative. As a rule, they involved:

- solving special types of polynomial equations;
- determining some term in the expansion of a binomial raised to a power;
- solving a word problem on deriving a quadratic equation (Shreider, 1937, p. 52).

The reduced number of topics, by comparison with the pre-Revolutionary years, is explained by the fact that the exam was supposed to cover only the curriculum of the preceding year; thus, many topics that had been studied in previous years were automatically excluded from the exam. After WWII, as contemporary mathematics educators noted (Matematika, 1948), exam problems became more difficult and thematically more diverse. Naturally, a standard exam prepared for students across the entire country had no room in it for problem types that were not specifically recommended for inclusion (such as composite problems). For example, here is the mathematics exam from 1948:

1. Two crews of workers were laying railroad ties on a railroad track. The first crew worked t more days than the second and covered s kilometers of the track with ties. The second crew covered m kilometers of the track more per day than the first crew, and over the course of its work it covered n kilometers of the track fewer than the first crew. How many kilometers of the track did each crew cover per day?
2. Define the constant term \( a \) in the polynomial \( 3x^5 - 4x^3 + 5x^2 + 7x + a \) in such a way that this polynomial becomes divisible by \( x+1 \) (without a remainder).

3. Determine for which values of \( a \) the fraction \( \frac{5a-4}{6-a} \) is greater than 1.

4. Calculate:
\[
\frac{8}{18} + \frac{5}{15} + \frac{4}{20} + \frac{3}{13} + 23 \cdot \frac{3}{16} + 19.116 + 2.7 + \frac{4}{5} - 2.4
\]
\[
5.76 \cdot 4.75 + 0.3 + 11 \cdot \frac{5}{7} - 3 \cdot \frac{1}{2}
\]

**On the Requirements for Writing Solutions on Exams**

This analysis of exam problems would be incomplete, however, without a discussion of the specific ways in which students were required to write the solutions to problems. Requirements for formulating solutions in writing were quite strict both before and after the Revolution. Students were required not only to solve the problems, but also to explain their solutions in detail, in other words, not simply to apply some method, but also to justify its application. Professors’ reports about exams are filled with complaints about solutions that are not adequately grounded (e.g. see the already cited Suslov, 1909, and Zimmerman, 1913). Furthermore, all notations had to be impeccable both in terms of their mathematics, and in terms of the Russian language. Suslov, for example, expresses indignation when students write “let us solve the binomial” instead of, say, “let us solve the problem on the application of the binomial theorem,” while Zimmerman diligently makes a note of every spelling error he finds.

Professors’ reports were intended to provide teachers with models of standards and requirements (and to punish those who failed to follow them). Exams were checked and graded differently in different gymnasia, and the level of detail that was considered acceptable in the explanations of solutions to problems varied from school to school. Zimmerman (1913) complained that at certain institutions students could receive even the highest grade while providing virtually no grounding or justification for their solutions. Nonetheless, teachers—and consequently their students—were by and large oriented toward solutions that were exhaustively grounded and formulated with the greatest degree of detail.

If in the 1930s lack of specificity concerning the content of exam solutions could still be tolerated, then by the second half of the 1940s the vagueness of such a general orientation had become unacceptable. For this reason, there appeared an instructional article by Gurvits and Filichev (1947) concerning standards and requirements for written problems in mathematics. This article provided teachers and students with explicit directions for writing solutions to problems, and it would be no exaggeration to say that these directions were based on something like a presumption of guilt on the part of the students. It was assumed that students had to prove in each and every step of each problem that they understood precisely what they were doing and writing (even when it came to
the simplest operations). For example, the authors recommend that the final answer to the solution of the inequality \( 4x^2 + 16x + 7 > 0 \) be written as follows (and we emphasize that the authors are talking only about the answer here—it was supposed to be preceded by a detailed solution): “Given the expression \( 4x^2 + 16x + 7 \), if we replace \( x \) with any number lesser than \( -\frac{7}{2} \) (for example, \( -4, -5 \), etc.) or any number greater than \( -\frac{1}{2} \) (for example, \( 1, 4 \), etc.), we will obtain positive values” (p. 46).

The article, however, turned out not to be entirely satisfactory. First, in a number of cases the authors remained unable to provide a clear determination of what must be done, and limited themselves to discussing the relative desirability of various alternative approaches. This led to a certain amount of confusion and further debate (for example, about how to check the solutions of word problems, see Smirnov, 1949). Second, the authors were so completely focused on making their notations as precise as possible that they actually made a significant number of mistakes in mathematics, which made them subjects of criticism themselves, notably by the well-known mathematician Yakov Dubnov (“O trebovaniyakh,” 1947).

After giving a detailed description of Gurvits’s and Filichev’s mistakes, Dubnov paints a portrait of their conception of the ideal student:

To convey their notion of the model student, the authors present us with a picture of a young person who has a good knowledge of formulas and techniques for solving standard problems, and an even better grasp of the laborious forms in which solutions must be written out, but (1) is unsure of his “mathematical rights,” (2) lacks a sense of initiative, (3) blindly follows rules, (4) is prepared to do unnecessary work for the sake of “form,” (5) is weak in logic, (6) is incapable of taking a critical approach to the premises of a problem, and (7) prone to dishonesty (“what I can’t explain, I’ll smooth over”) (“O trebovaniyakh,” 1947, p. 54).

Dubnov then poses the question: what kind of person do we want our educational system to produce—someone who carries out instructions to the letter or someone capable of critically making sense of the work assigned to him?

The debate was summed up and concluded by Pavel Larichev, a Ministry official and textbook author, in an article in 1948. After critiquing the excessive wordiness of the solutions in Gurvits’s and Filichev’s article (1947), and after acknowledging their mistakes, he essentially asserted that insistence on extremely detailed explanations was the proper course to take. Somewhat later this principle was expressed as follows: “The solution to certain problems should be accompanied by a detailed textual explanation, which should in essence constitute an essay on a mathematical topic” (Printsev, 1951, p. 72).
Larichev remarked that it was wrong to rely on instructions that students could always expect to apply automatically (in doing so, he contrasted Soviet methods with those of teachers abroad, who believed in testing as a supposedly objective form of measurement). As a result, many teachers did not know how to anticipate what the authorities would consider to be wrong in the future and strove to safeguard themselves. Thus, the Leningrad teacher Ginzburg (1949, p. 22) admitted that he would lower grades not because he believed that a mistake had been made, but because he was afraid that what his students had done would be considered a mistake by his supervisors.

**Discussion and Conclusion**

Exam requirements and concrete exams in mathematics are part of the culture and politics of an age and therefore contribute to an understanding of it. Dubnov (“O trebovaniyakh,” 1947), who examined what would appear to have been purely mathematical requirements for formulating solutions, discerned in them an image of the kind of citizen that it would be desirable for the education system to produce. It may be argued that in making such a connection, he was essentially right. Naturally, no one ever formulated such a link explicitly while writing exam problems or formulating methodological recommendations; and it is unlikely that the authors criticized by Dubnov would have agreed that this was indeed how they had envisioned the ideal product of the Soviet school system. But the specific mentality that is endorsed and promoted by the state and society is reflected in practically everything—not for nothing did the classic works on cultural history rely, for example, on the analysis of the fine arts of the epochs they studied (Huizinga, 1996). Even taking into account all of the differences between art and mathematics education, we would nonetheless reiterate that the analysis of exam problems also helps to form a better understanding of the people of each epoch. For example, the fact that students taking the gymnasium exam in algebra were required to demonstrate precision (as described by Markovich, 1913) and a certain degree of competence—in the sense of being able to understand what they were told and to respond in a meaningful and informed manner—makes it easier to visualize the contemporary conception of the ideal gymnasium graduate as a potential future functionary in an enormous, illiterate country governed by a bureaucratic machine. Likewise significant for an understanding of historical change in Russia—including changes in its education—is the transformation in attitudes toward the possibility of successfully passing an exam even while making mistakes in certain isolated problems.

Turning to the issue of the practical analysis of historical records, we would note that it is naive to hope to find in history ready answers to today’s questions; nor is it always possible to find clear-cut conclusions about the relatively effectiveness of various measures. Despite numerous critical publications about graduation exams in gymnasia—and in particular about their unreliability for assessing student knowledge—the professors of Moscow University, when surveyed in 1899 (Soveschaniya, 1899), for the most part responded in the negative...
when asked whether it would be desirable to establish a special exam for admittance to the university, thus attesting to a sufficiently high level of objectivity in the gymnasium exams.

The picture that emerges before us is complex, and this is arguably the most important finding. The education system changes extremely slowly—the reappearance of composite problems on exams in the 1930s is a vivid illustration of this fact. The country changed, society changed, education was faced with entirely new challenges, but the writers of problems continued trying to do everything as in the good old days.

Centralization was often seen as a rapid means of overcoming that which had become out of date, and more generally, of creating order (among other things, by offering everyone equal opportunities, the lack of which had been deplored by Zimmerman). It turns out, however, that centralization and micromanagement can also increase confusion, even to the point of provoking people to break with and deviate from established procedures, in ways that the Ministry of Education simply could not control, even if it wished to do so. Centralization vs. local decision-making is one of the fundamental oppositions that we observe in the history of exams, with each side having its own strengths and shortcomings.

Directly using exams (with students’ grades and the standards by which their problems were graded) to assess the effectiveness of education at a school may, as we have seen, lead teachers “not to notice” cheating—which calls into question their use for this purpose as well as their use to assess individual students. In general, how exam results can be used and what kinds of information can be derived from them is an important and persistent problem.

Even a century ago, mathematics educators already recognized the danger of teaching students nothing but skills and of testing only how well students can follow a fixed pattern. Their real goal was to teach students how to think. Convoluted constructions, along with a heightened demand for explanations, were seen as means for fighting against the use of fixed patterns and the merely formal memorization of algorithms. The ability to read (untangle) a problem and to write out every single premise in its solution was equated with understanding. Meanwhile, as we have seen, these methods in the struggle against fixed patterns themselves turned into fixed patterns that teachers taught their students, instead of developing their comprehension and cultivating their creativity.

The controversy concerning examinations in Russia is not over. In the winter of 2006, the Russian federal legislature took up the issue of introducing fundamental reforms to the graduation exam system. Meanwhile, such examinations, along with the attendant controversies, continue in other countries as well. It appears valuable and important to study the history of the examinations everywhere. Such studies could allow educators to become thoroughly familiar with the ways in which such exams were conducted in the past, and thus to recognize the challenges and contradictions that might arise, and the difficulty of resolving them.

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Acknowledgements
The author wishes to express his gratitude to Patricia Jendraszek for her assistance in preparing the manuscript.

Notes
1. All translations from Russian are by the author.

References

Berezanskaya, E.S., Margulis, A.Ya. (1938). Pis’mmennye kontrol’nnye ispytaniya v X klasse vesnoy 1937 goda. (Written Exams in Tenth Grade in the Spring of 1937.) Matematika v shkole, 3, 60-68.


Iz otchetov o pis’mennykh rabotakh, vypolnennykh na ispytaniakh zrelosti v gimnaziyah v 1896 g. (Excerpts from reports on written solutions provided on graduation exams at gymnasia in 1896.) (1897). *Tsirkuliar po Sankt-Peterburgskomu uchebnomu okrugu*, 433-445.


Karp, A. (1998). *Pis’mennye vypusknye eksameny po algebre v Rossii sa 100 let* (Russian Written Examinations in Algebra over 100 Years.) St. Petersburg University of Education.


Nashi uchebnye zavedeniya. Ob ispytaniyakh zrelosti v 1907 g. (Our educational institutions. On graduation exams in 1907.) (1908). Zhurnal Ministerstva narodnogo prosvescheniya, December, 47.

Ob ekzamenakh na attestat zrelosti i vypusknykh ekzamenakh za kurs nachal’noy i semiletney shkoly. (On exams to obtain a diploma and graduation exams for elementary and seven-year schools.) (1945). Moscow: Uchпедgиз.

O poriadke vybora tem v srednikh uchebnykh zavedeniakh dlia ispytaniy zrelosti. (Procedures for selecting topics for exams in high school graduation exams.) (1912). Tsirkuliar po Sankt-Peterburgskomu uchebnomu okrugu, 12, 482-484.

O predvaritel’nykh itogakh vesennikh ispytaniy 1932/33 uchebnogo goda v nachal’noy i sredney shkolakh. (On the preliminary outcomes of the Spring exams of the 1932-33 school year in elementary and secondary schools.) (1933). Biulleton’ Narodnogo Komissariata po prosvescheniu RSFSR, 15, 3.


Po voprosu o kharaktere zadach, predlagaemykh po matematike na pis’mmenykh vypusknynkh ispytaniyakh v srednikh uchebnykh zavedeniakh. (On the nature of the problems offered on high school graduation exams in mathematics.) (1912). Tsirkuliar po Sankt-Peterburgskomu uchebnomu okrugu, 144-145.

Polozhenie o edinoy trudovoi shkole RSFSR. (Ordinance concerning a universal labor school in the RSFSR) (1918). In Sbornik dekretov i postanovlenii rabochekrest’ianskogo pravitel’stva po narodnomu obrazovaniiyu. Issue I. Moscow, 107-112.

Pravila ob ispytaniyakh uchenikov gimnazii i progimnazii vedomstva Ministerstva narodnogo prosveshenia. (Rules for Testing Students of Gymnasia and Pre-Gymnasia affiliated with the Ministry of Education). (1891). Tsirkuliar po Sankt-Peterburgskomu uchebnomu okrugu, 203-244.


Tirpotin, A.D. (1912). Otchet o pis’mennykh rabotakh po matematike, ispolnennykh uchenikami gimnazii i real’nykh uchilishch Kavkazskogo uchebnogo okruga i postoronnimi litsami na okonchatel’nykh ispytaniakh v 1911 g. (Report on Written Answers Given by Students of Gymnasia and Vocational Schools in the Causcasus School District and Other Individuals on Graduation Exams in Mathematics.) Tiflis.


Zimmerman, V.A. (1913). Otzyv o pis’mennykh rabotakh po matematike, ispolnennykh na okonchatel’nykh ispytaniakh v real’nykh uchilishchakh Odesskogo uchebnogo okruga

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